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# A new cosmological constant model

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## Abstract

We propose a new cosmological model with a time-dependent cosmological constant ( $\Lambda \propto 1/t^2$ ), which starting at the Planck time as  $\Lambda_{Pl} \sim M_{Pl}^2$ , evolves to the present-day allowed value of  $\Lambda_0 \sim 10^{-120} M_{Pl}^2$ . This scenario is supported by non-critical string theory considerations. We compute the age of the Universe and the time-dependence of the scale factor in this model, and find general agreement with recent determinations of the Hubble parameter for substantial values of  $\Omega_\Lambda$ . This effectively low-density open Universe model differs from the traditional cosmological constant model, and has observable implications for particle physics and cosmology.

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The problem of the cosmological constant ( $\Lambda$ ) has been with us for over 75 years, ever since Einstein introduced it in order to avoid an expanding Universe. Of course, Einstein was misinformed, although on the other hand, since “whatever is not forbidden is mandatory” one can say that he acted in a physically reasonable way. A lot of effort has been spent tackling this monumental problem, and yet a universally acceptable solution is still lacking. The basic problem is that the vacuum energy of (spontaneously broken) gauge quantum field theories, when coupled to gravity is metamorphosed into a cosmological constant which is usually much larger than the *presently* allowed one.

In this short note we propose yet another cosmological constant model. While our model contains seeds of some ideas that have been developed recently in the framework of non-critical string theory [1], it is motivated by more general, phenomenologically oriented principles, as well as by the recent determination of the Hubble parameter ( $H_0$ ) by the Hubble Space Telescope.

Let us start from the Einstein-Friedmann equation of standard Big-Bang Cosmology

$$H^2 = \frac{8\pi G}{3} \rho - \frac{k}{R^2} + \frac{\Lambda}{3} , \quad (1)$$

where  $H = \dot{R}/R$  is the Hubble parameter,  $R$  is the scale factor,  $G$  is Newton’s constant,  $\rho$  is the particle energy density, and  $k = +1, -1, 0$  is the curvature parameter for a closed/open/flat Universe. The present day values of the various parameters ( $H_0 \sim 1/t_0 \sim 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , with  $t_0 \sim 10 \text{ Gyr}$  the age;  $T_0 \approx 2.73 \text{ K}$ ,  $\Lambda_0 \lesssim 10^{-120} M_{Pl}^2$ ) are extremely small or large, and make the standard Big-Bang Cosmology rather unnatural. Inflation has been introduced to solve some of these problems with great success: an “instantaneous” blow up of the scale factor ( $R_{\text{after}} \sim 10^{28} R_{\text{before}}$ ) creates a smooth, effectively “flat” (*i.e.*,  $k/R^2 \rightarrow 0 \Leftrightarrow k_{\text{eff}} = 0$ ) Universe of the right size and entropy to describe our Universe. This natural obliteration of the curvature term is rather effective, and it is interesting to see whether the same can be done to the cosmological “constant” term. Let us suppose that

$$\Lambda = \frac{\Lambda_{Pl}}{(R/\ell_{Pl})^\alpha} , \quad (2)$$

where  $\Lambda_{Pl} \sim M_{Pl}^2$  is the “natural” size of the cosmological constant,  $(R/\ell_{Pl})$  is the scale factor in units of the Planck length, and  $\alpha$  is a constant to be determined by the present upper bound on  $\Lambda$ :  $\Lambda_0 \lesssim 10^{-120} M_{Pl}^2$ , and the present value of  $R$ :  $R_0/\ell_{Pl} \sim ct_0/\ell_{Pl} \sim 10^{60}$ . That is  $\alpha = 2$ , and

$$\Lambda = \frac{\Lambda_{Pl}}{(R/\ell_{Pl})^2} \propto \frac{1}{R^2} , \quad (3)$$

has the same  $R$  dependence as the curvature term, and would give us today an acceptable and perhaps even detectable cosmological “constant”. Let us emphasize the effect of inflation by rewriting Eq. (1) as  $H^2 \sim M_{Pl}^2 (T/M_{Pl})^4 + M_{Pl}^2 (\ell_{Pl}/R)^2$ . Before inflation  $T \sim M_{Pl}$ ,  $R \sim \ell_{Pl}$  and both terms are comparable; after inflation and

reheating  $T \sim M_{Pl}$ ,  $R \sim 10^{28} \ell_{Pl}$  and the cosmological constant term is very small compared to the radiation term. In the subsequent adiabatic expansion  $R \propto 1/T$  and we can write  $H^2 \sim M_{Pl}^2 (T/M_{Pl})^4 + M_{Pl}^2 10^{-56} (T/M_{Pl})^2$ . This relation shows that the cosmological constant term becomes comparable to the radiation term at  $T \sim 10^{-28} M_{Pl} \sim 1 \text{ eV}$ , *i.e.*, the usual early Universe cosmology is undisturbed.

It is also useful to give the time dependence of  $\Lambda$ , since this avoids keeping track of the rapid change in  $R$  during the inflationary phase. An ansatz similar to that in Eq. (2) gives

$$\Lambda = \frac{\Lambda_{Pl}}{(t/t_{Pl})^2} \propto \frac{1}{t^2}, \quad (4)$$

implying that  $R \propto t$  somehow (this relation is only valid at late times; in the early Universe  $R \propto t^{1/2}$ ).

The above analysis is purely phenomenological, but suggests strongly that we should seek a fundamental theory which can produce such  $R$  or  $t$  dependence of the cosmological constant. In fact, an example of such a theory in the context of non-critical string theory already exists in the literature. In Ref. [2] it was found that string theory admits only a few cosmologically interesting (*i.e.*, expanding Universe) solutions, all leading asymptotically to a “vacuum” characterized by

$$R \propto t, \quad \Lambda \propto e^\Phi \delta c \propto 1/t^2 \propto 1/R^2, \quad (5)$$

where  $\Phi$  is the dilaton field, and the central charge deficit  $\delta c \neq 0$  reflects the departure from criticality and gives rise to the cosmological constant  $\Lambda$ . Moreover, the curvature (Ricci) scalar  $\mathcal{R} \propto 1/t^2$ , and in suitable units  $\mathcal{R} = \Lambda$ . That is, even in a flat universe ( $k = 0$ ) we have an effective curvature which “fakes” the cosmology of an *open* Universe, as we discuss below. In other words, the detailed string equations of motion for this cosmological vacuum reduce, sufficiently below the Planck scale, to an *effective* Einstein-Friedmann equation (1) with the phenomenologically desired  $R$ -dependent cosmological “constant”.

It should be stressed that the above cosmological string vacuum should be viewed as the correct asymptotic vacuum of string theory, just as the Minkowski vacuum is to ordinary point-particle quantum field theory. As such, we expect the above discussed ( $R$ -dependent) relations to hold in any realistic string vacuum where the full gravitational and matter multiplets are accounted for. The picture presented in Ref. [2] has been completed by the addition of string dynamics in Ref. [3], where it was argued that not only the cosmological “constant”, but all “constants” in nature (*e.g.*,  $c, \hbar$ ) become time dependent. Moreover, the desired initial conditions  $\Lambda_{Pl} = M_{Pl}^2$  and  $k = 0$  are derived in this dynamical scenario [3]. Also, in this dynamical scenario one obtains  $\Lambda \propto 1/R^2$  without an explicit reference to a time-dependent dilaton field [3]. This is important since, as is, our toy model above entails a possibly troublesome time-dependent gauge coupling ( $g \propto e^{\Phi/2}$ ). Our present phenomenological approach will be incorporated in the more rigorous string dynamics picture of Ref. [3] elsewhere [4].

We now elaborate on the observational consequences of our cosmological model. Starting from Eq. (1), one can easily determine the age of the Universe in the matter-dominated era, with  $\rho = \rho^M = \rho_0^M (R_0/R)^3$  and  $\Lambda = \Lambda_0 (R_0/R)^2$  (also  $k = 0$ ). In our flat Universe model  $\Omega_M + \Omega_\Lambda = 1$  (where  $\Omega_M = \rho_0^M/\rho_0$ ,  $\Omega_\Lambda = \Lambda_0/3H_0^2$ , and  $H_0^2 = \frac{8\pi}{3}G\rho_0$ ) and we get

$$t_0 = H_0^{-1} f(\Omega_\Lambda) , \quad \text{with} \quad f(x) = \frac{1}{x} - \frac{1-x}{x^{3/2}} \ln \left( \frac{1+\sqrt{x}}{\sqrt{1-x}} \right) . \quad (6)$$

This result has the property that  $t_0$  is finite (*i.e.*,  $f(x) \leq 1$ ) for all values of  $\Omega_\Lambda \leq 1$ . For comparison, the traditional cosmological constant model, where  $\Lambda = \Lambda_0$  is truly constant, gives

$$t_0 = H_0^{-1} g(\Omega_\Lambda) , \quad \text{with} \quad g(x) = \frac{2}{3} \frac{1}{\sqrt{x}} \ln \left( \frac{1+\sqrt{x}}{\sqrt{1-x}} \right) , \quad (7)$$

which diverges for  $\Omega_\Lambda \rightarrow 1$ . In numbers, the age of the Universe is  $t_0 = \frac{1}{h}(9.8 \text{ Gyr}) f(\Omega_\Lambda)$ , where we take the recent Hubble Space Telescope determination of  $h = 0.80 \pm 0.17$  [5]. The results for various values of  $h$  and both cosmological constant models are shown in Fig. 1. We note that for sufficiently high values of  $\Omega_\Lambda$ , both models are generally consistent with the estimated age range of  $14 \pm 2 \text{ Gyr}$ , although  $\Omega_\Lambda$  needs to be larger in our new model. For instance, if  $h = 0.8$ ,  $t_0 > 10 \text{ Gyr}$  for  $\Omega_\Lambda \gtrsim 0.7$  in our model, whereas  $\Omega_\Lambda \gtrsim 0.45$  would do in the traditional model.

By integrating the Einstein-Friedmann equation up to an arbitrary time  $t$ , we can also determine the time-dependence of the scale factor, which for an “empty” (although “filled” with a  $t$ -dependent cosmological “constant”) Universe is supposed to be  $R \propto t$ . We obtain

$$\sqrt{\Omega_\Lambda} t H_0 = \sqrt{x(a+x)} - a \ln \left( \frac{\sqrt{x} + \sqrt{a+x}}{\sqrt{a}} \right) , \quad (8)$$

where  $x = R/R_0$  and  $a = (1 - \Omega_\Lambda)/\Omega_\Lambda$ . For  $\Omega_\Lambda = 1$  this reduces to  $tH_0 = R/R_0$  as anticipated. In contrast, the result in the traditional cosmological constant model is

$$\sqrt{\Omega_\Lambda} t H_0 = \frac{2}{3} \ln \left( \frac{\sqrt{x^3} + \sqrt{a+x^3}}{\sqrt{a}} \right) , \quad (9)$$

which for  $\Omega_\Lambda \rightarrow 1$  gives  $R \sim e^{H_0 t}$  as expected. The  $t$ -dependence of the scale factor for various values of  $\Omega_\Lambda$  is shown in Fig. 2.

Since the age of the Universe constraint appears to require a substantial value of  $\Omega_\Lambda$  (*e.g.*,  $\Omega_\Lambda \gtrsim 0.7$ ), in our flat model the matter relic abundance will need to be suppressed (*e.g.*,  $\Omega_M \lesssim 0.3$ ). This situation would appear to be similar to the traditional cosmological constant model studied in the literature [6] which has some advantages and some disadvantages. However, this is not so, since in our case we

effectively have a low-density open universe, with different (and perhaps more desirable) properties regarding structure formation [7]. Moreover, the particle physics contribution to the relic density ( $\Omega_M$ ) can be easily accommodated in a supersymmetric model with say  $\Omega_M h^2 \lesssim 0.3(0.8)^2 \sim 0.2$ , as explored previously in Ref. [8].

In sum, we have proposed a new cosmological model with a “running” cosmological constant which, starting at the Planck time at its natural value, evolves with time as  $1/t^2$  and attains a present-day value in agreement with observations. This model is supported by non-critical string theory considerations, which also provide a justification for the initial conditions used. Our effectively low-density open Universe model should be testable in ongoing particle physics experiments as well as through cosmological observations of the age and structure of the Universe. Curiously enough, if indeed it holds true that the cosmological constant is both non-zero and time-dependent, it would fulfill Dirac’s prophesy that there are no small constants, simply the Universe is too old [9], and at the same time it would redeem Einstein from what he thought was the biggest blunder of his life, *i.e.*, the introduction of the cosmological constant, leaving only the very minor blunder of thinking that a cosmological constant different from zero was indeed a blunder!

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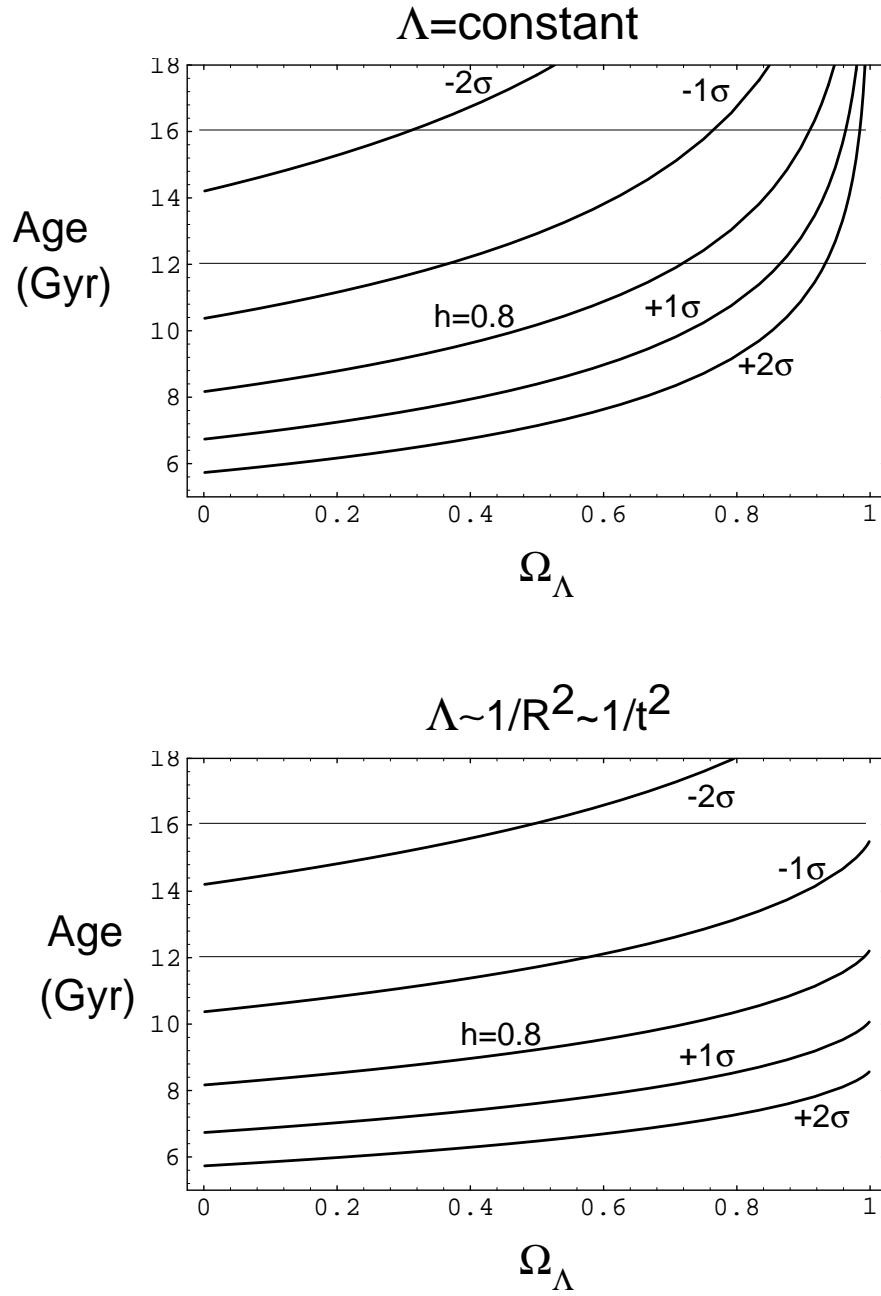


Figure 1: The calculated age of the Universe (in Gyr) as a function of  $\Omega_\Lambda$  in the traditional ( $\Lambda = \text{constant}$ ) and our new ( $\Lambda \propto 1/R^2 \propto 1/t^2$ ) cosmological constant model, for various choices of the Hubble parameter ( $h = 0.80 \pm 0.17$ ). The standard Big-Bang Cosmology prediction (for  $\Omega_0 = 1$ ) is recovered for  $\Omega_\Lambda = 0$ . The horizontal lines delimit the estimated range for the age of the Universe ( $14 \pm 2$  Gyr).

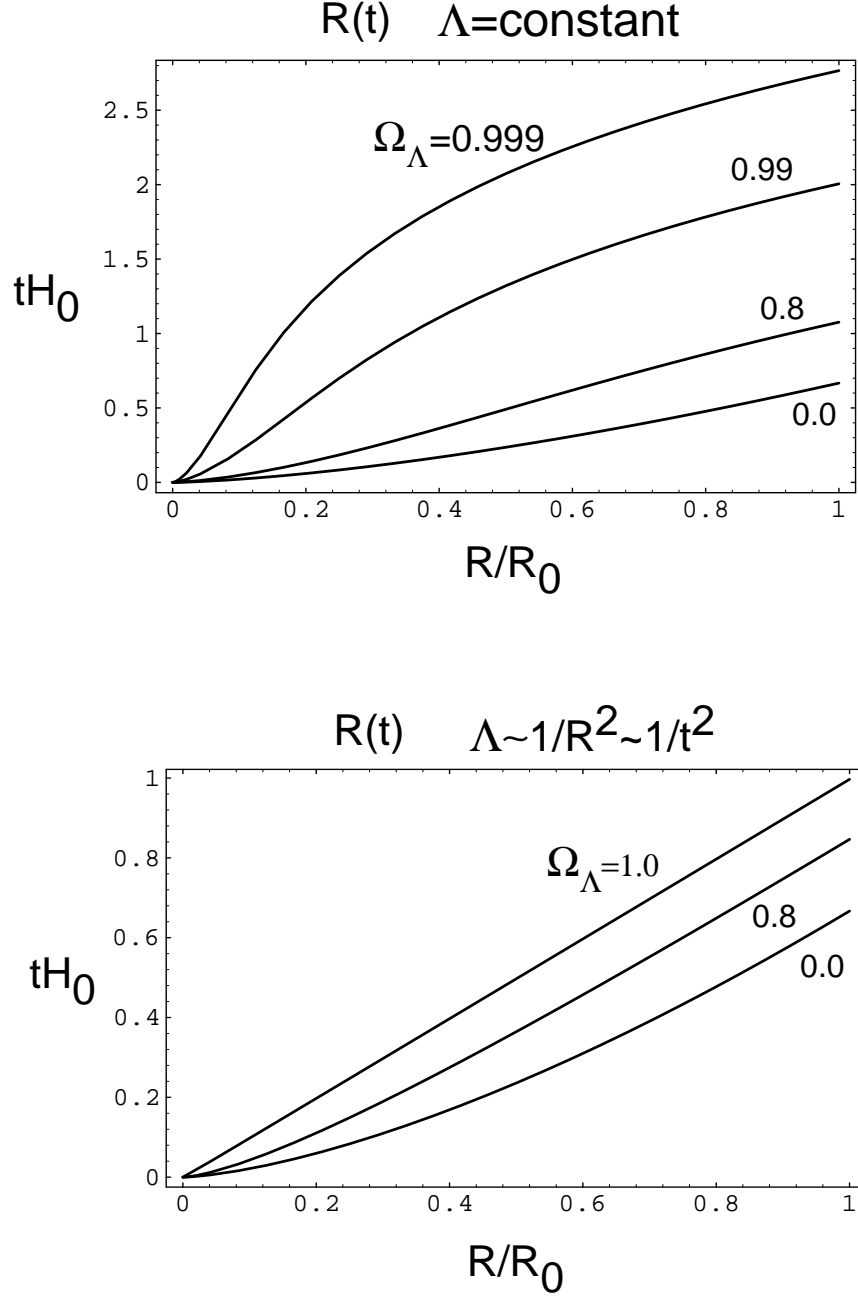


Figure 2: The time dependence of the scale factor  $R$  in the traditional ( $\Lambda = \text{constant}$ ) and our new ( $\Lambda \propto 1/R^2 \propto 1/t^2$ ) cosmological constant model, for various values of  $\Omega_\Lambda$ . Note that  $R(t) \propto t$  in the new model for  $\Omega_\Lambda \sim 1$ .